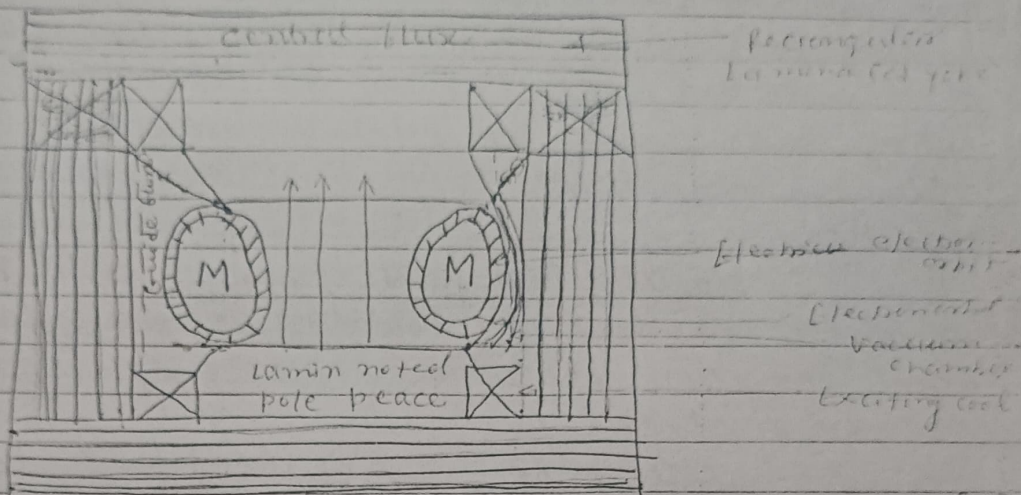


Betatron is a cyclic accelerator for accelerating a continuous beam of electron of high speed by mean of electric field produced by changing the magnetic flux. The electron move in a stable circular orbit, evaluated donut nut shaped chamber. Betatron is capable of producing 300 MeV energy which is the largest and best for ultimate.



[ Fig - (a) Vertical section ]

Construction: → It consists of dough nut shaped vacuum chamber placed in between the pole pieces of a very strong magnet powered by an alternating pulse current. The magnetic field performs several functions.

- ① The field by exerting Lorentz force guides the electron to move in a circular orbit inside the doughnut shaped vacuum chamber.
- ② It accelerates the electron in circular orbit.
- ③ The field varies in the fashion that the condition  $mv = \text{constant}$  for constant radius is satisfied i.e. change of momentum is balanced by varying  $\beta$ .
- ④ It provides the restoring force which resist the tendency of the electron beam to leave their orbit radially or vertically.
- ⑤ It help to introduce the electron into the orbit initially and when they obtain maximum energy helps in their extraction from the orbit.

All their functions are accomplished by a non shaping and control of magnetic field.

The doughnut shaped vacuum chamber is made

glass and its inner side coated with a thin layer of silver to avoid the accumulation of surface charges. To avoid the eddy currents the pole pieces of laminated iron and yoke is also in the laminated form.

Electrons are injected into doughnut at about 90 kV by a pulsed electron gun placed near the orbit and aimed tangentially.

Working principle --- The action of betatron depends upon the same principle as that of transformer. Here primary current is the current following in exciting coil of magnet and secondary is the coil of electron circulating in doughnut. The primary current produces an oscillating magnetic field which in turn induces accelerating the electrons.

The condition that magnetic field guides the electrons and accelerates them. According to Faraday's law of e.m.f induction the work done on an electron of charge "e" in revolution is

$$W = e \cdot \frac{d\phi}{dt} \quad (\phi = \text{flux})$$

If  $F$  be the tangential force acting on the electron. The work can be expressed as

$$W = F \times 2\pi r \quad (r = \text{radius of the orbit})$$

$$F = \frac{e}{2\pi r} \frac{d\phi}{dt} \quad \text{--- (1)}$$

For the stable orbit

$$\beta e v = \frac{m v^2}{r} \quad \text{or } m v = \beta e r \quad \text{--- (2)}$$

The equation (2) indicates that the magnetic field  $\beta$  at the orbit must increase as the electron energy increases otherwise radius  $r$  will not be constant. If  $r$  is kept constant, then

$$\frac{d}{dt} (m v) = e \cdot r \cdot \frac{d\beta}{dt} \quad \text{--- (3)}$$

From Newton's second law, the rate of change of momentum is equal to the force. Hence from the equation (1) and (3), we get

$$\frac{d}{dt} (m v) = \frac{2\pi r}{e} \times e \times r \times \frac{d\beta}{dt} = 2\pi r^2 \frac{d\beta}{dt} = 2 \times \frac{d}{dt} (r^2 \beta) \quad \text{--- (4)}$$

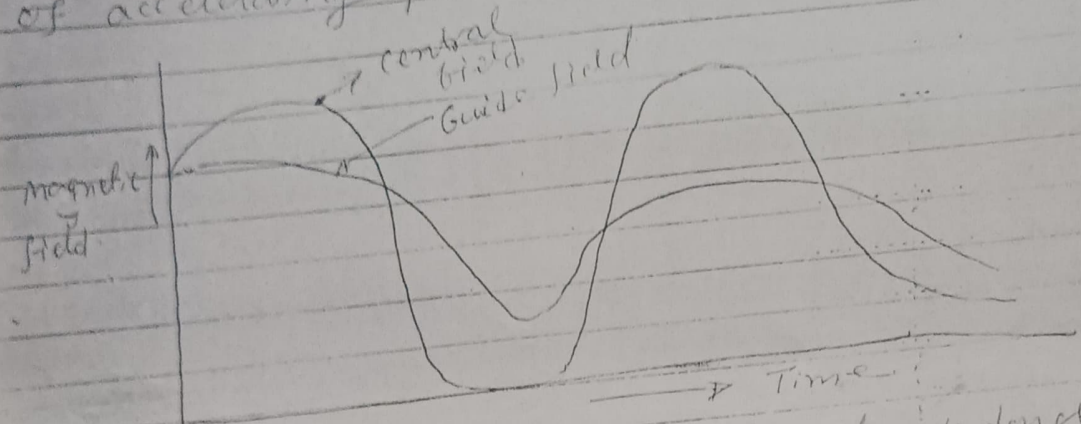
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Now when  $B_0$  is the mean field strength with the orbit then (4) can be written as

$$\frac{dB_0}{dt} = 2 \times \frac{dB}{dt} \quad (5)$$

The equation (5) is known as Betatron condition and states that the average rate of change of flux density with in the orbit  $\frac{d\Phi}{dt}$  must be equal to twice the rate of the orbit  $\frac{dB}{dt}$ .

Operation — The variation of magnetic field with time due to a.c pulse energizing the magnet as shown in fig. the changing magnetic field induces an e.m.f. of accelerating phase in the beginning of an cycle.



If the electron are ejected into doughnut in the beginning of the cycle phase of e.m.f they are accelerate for quarter cycle. After a certain time they obtain maximum energy corresponding to the cana at point P, a pulse current is sent through a coil which suddenly change the magnetic field and high energy electrons are displaced from the stable orbit and are used by focussing into a target.

Energy of accelerated particles — Let control flux variation be given by  $\phi = \phi_0 \sin \omega t$ . The acceleration takes place for quarter cycle. Hence energy gained per atom is

$$e \cdot E \cdot \pi r^2 = e \cdot \frac{d\phi}{dt} \cdot \pi r^2 = e \cdot \omega \phi_0 \cos \omega t \cdot \pi r^2$$

The average energy per turn will be

$$= \frac{e \omega \phi_0 \int_0^{\pi/2} \cos \omega t \cdot dt}{\int_0^{\pi/2} dt} = \frac{e \omega \phi_0 \left[ \frac{\sin \omega t}{\omega} \right]_0^{\pi/2}}{\pi/2} = \frac{2 \omega e \phi_0 r^2}{\pi}$$

The acceleration of electrons have a velocity very close to that of light. Hence the total distance travelled by an electron during quarter cycle  $\frac{\pi}{2\omega}$  and the distance travelled while being in equilibrium orbit of radius  $r = 0.200$

It is the no. of revolutions making by the electron to covering the distance  $\frac{c\pi}{2\omega}$  in the time  $\frac{\pi}{2\omega}$  is given by

$$N \times 2\pi r = \frac{c\pi}{2\omega} \Rightarrow \frac{1}{N} = \frac{c}{4\omega r}$$

The total energy gained by the electron  
 = Energy gain per revolution  $\times$  no. of revolutions

$$E = 2e\phi_0\omega \times \frac{c}{4\omega r} = \frac{ec\phi_0}{2r}$$

A notable feature of this equation is that the energy is independent of the angular frequency  $\omega$  of the charging magnetic flux.

Radiation loss: — The radiation loss  $\Delta E$  for electron per cycle is

$$\Delta E = \frac{8.39 \times 10^{-37}}{r} \left(\frac{E}{c}\right)^4 \text{ eV / revolution}$$

where  $E/c$  represents energy in  $E_0$  and it has been assumed that  $E \gg m_0c^2$

Maximum Energy Limit: — As the operation of this is independent of relativistic increase of mass it appears that externally with energy might be obtained with the help of this device. But this not much because radiation loss increases as the fourth power of energy gain and this sets of the high energy limit upto 300 MeV

Practical limit: — (i) It is bulky in size.  
 (ii) Because of the large layer air gap between the reactive power runs into thousands of kilowatt and to minimize it betatron is connected in capacitor bank found to incidence of the magnet.

(iii) Small part of magnet is used to provide guide field which the large portion of Bulky magnet remain functionless.

(iv) Higher energy limit upto 300 MeV.